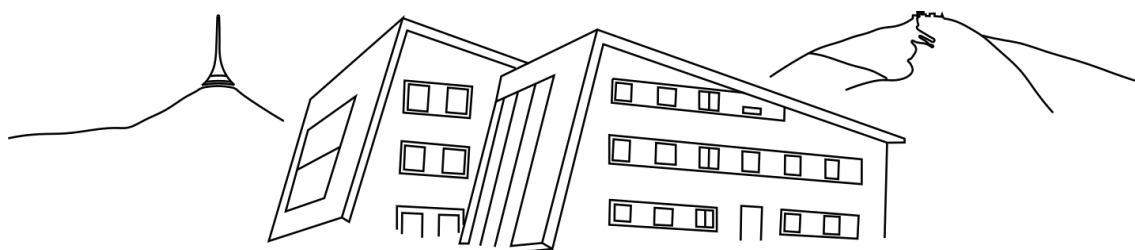


Spring School on Analysis 2025

Function Spaces and Applications XIII



Book of Abstracts



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Invited lecture courses

Bernhard Ruf

University of Milan

Exponential growth in 2-d: Function spaces, optimal embeddings and critical PDE's

Abstract: For domains $\Omega \subset \mathbb{R}^2$ the Sobolev space $H_0^1(\Omega)$ embeds into any L^p -space, and so one may look for a higher than polynomial growth for an optimal embedding. The Trudinger-Moser inequality states that there is an optimal embedding into the Orlicz-space $L_\varphi(\Omega)$, with maximal growth function $\varphi(s) = e^{s^2}$. We will discuss various forms of related inequalities.

In \mathbb{R}^N , $N \geq 3$, the best Sobolev embedding constants are known, and are never attained for bounded domains. To the contrary, in dimension 2 the best embedding constant is not known and, surprisingly, it was shown by Carleson-Chang (1986) that it is attained (if Ω is a disk). This implies that the associated Euler-Lagrange differential equation has a solution. We will present various existence results for related elliptic equations, which all present the difficulty of *loss of compactness* (see e.g. de Figueiredo-Miyagaki-R., 1995).

The fact that the supremum in the TM-inequality is attained lead Struwe (1988) to suspect, and prove, that the global maximum “survives” as a local maximum in the slightly supercritical case (in which the global supremum becomes infinite). This in turn lead to the search of mountain-pass solutions in the slightly super-critical case, which *blow up* as the super-critical exponent approaches the critical exponent from above (del Pino-Musso-R., 2010).

Beside the H^1 -solutions (which are in fact regular), elliptic equations with exponential nonlinearities admit also *singular* solutions. They are *distributional* solutions which lie just barely outside of H^1 (and hence are not weak solutions). We discuss the construction of such solutions, and then show that they give rise to *non-uniqueness* for the associated heat equation (Fujishima-Ioku-R.-Terraneo, 2024).

Armin Schikorra

University of Pittsburgh

Topological obstructions for Sobolev spaces

Abstract: We will delve into various aspects of topological obstructions within the framework of Sobolev spaces. To illustrate fundamental principles, we will initially explore a Sobolev adaptation of the Brouwer Fixed Point theorem. This exploration will naturally lead us to considerations regarding the definition of degree for Sobolev maps between manifolds. Subsequently, we will examine Sobolev maps with restricted rank, alongside examples illustrating topological obstructions encountered in the approximation or extension of Sobolev maps such as homeomorphisms. It is assumed that participants are acquainted with the theory of Sobolev spaces in Euclidean contexts. The topological concept needed will be defined throughout the course.

Nageswari Shanmugalingam

University of Cincinnati

Short course on first order analysis in nonsmooth spaces using upper gradients

Abstract: In this course we will give an introduction to Sobolev space-theory for non-smooth metric measure spaces where the notion of weak derivatives is unavailable. We will discuss the notion of upper gradients inspired by Riemannian geometry, and discuss geometric properties of metric measure spaces of bounded geometry.

Lecture 1: p -moduli of families of curves, and ACL_p condition. In this lecture, after a brief introduction to path integrals, we will discuss notions of p -modulus of families of curves. We will then discuss the Euclidean setting, and the property of *absolute continuity on p -modulus almost every line segment* parallel to the coordinate axes (ACL_p -property). We will also discuss the Beppo-Levi characterization of Sobolev functions.

Lecture 2: Upper gradients and their properties. In this lecture, we will discuss the notions of upper gradients and p -weak upper gradients of measurable functions on a metric measure space X . We will discuss what the analogs of the classical differentiation rules are in this setting, and construct Newton-Sobolev spaces $N^{1,p}(X)$. We will discuss the Banach lattice property of $N^{1,p}(X)$, and consider some example non-smooth spaces as well.

Lecture 3: Poincaré inequalities. In this lecture, we will talk about the main equivalence class property and Poincaré inequalities, and discuss what the geometric implications are of a metric measure space that supports a Poincaré inequality. In this lecture we will assume for the most part that the measure is doubling (I will explain what that means as well).

Lecture 4: Other candidate Sobolev spaces in non-smooth setting. In this lecture, we will discuss alternate function-spaces such as Hajlasz-Sobolev spaces, Poincaré-Sobolev spaces, Korevaar-Schoen Sobolev spaces, and time permitting, we will also discuss the theory of Dirichlet forms briefly.

Short talks

Riju Basak

National Taiwan Normal University

Hardy spaces associated with the twisted Laplacian

Abstract: Hardy spaces have played a fundamental role in harmonic analysis, complex analysis, and partial differential equations. They often serve as substitutes for L^p spaces when $0 < p \leq 1$ in various applications. A central theme in the study of Hardy spaces is the development of equivalent characterizations, such as those involving atomic decompositions or various maximal functions, which are crucial tools in establishing the boundedness of singular integrals and Fourier multiplier operators.

In this talk, I will discuss Hardy spaces associated with the twisted Laplacian and present several equivalent characterizations. If time permits, I will also present an application involving sharp estimates for the wave equation associated with the twisted Laplacian.

Mohd Ashraf Bhat

Indian Institute of Technology Ropar

Wiener-Lebesgue property for Sobolev functions on metric spaces

Abstract: The classical Lebesgue differentiation theorem states that almost every point of a locally integrable function defined on \mathbb{R}^n is a Lebesgue point. If the functions are associated with some regularity, namely they are Sobolev functions, then this theorem holds not only with a better integrability exponent but also with the exceptional sets are of zero capacity. In this talk, we will show that that first-order Sobolev functions defined on a complete, doubling metric measure space satisfy a Wiener-type Lebesgue point property. This property is stronger than the standard Lebesgue point property, except that the set of non-Lebesgue points is slightly larger.

Antoine Dettaille

Université Claude Bernard Lyon 1 – Institut Camille Jordan

Density problems for Sobolev maps into manifolds

Abstract: In a striking contrast with the situation for real-valued maps, Sobolev maps with values into a compact manifold \mathcal{N} *need not* be strongly approximable with smooth \mathcal{N} -valued maps. This observation, originally due to R. Schoen and K. Uhlenbeck (1983), has triggered an important amount of research on the properties of Sobolev spaces of mappings into manifolds, and some of the related questions are still widely open.

In this talk, I will try to present some of these questions, as well as a few key results in the area, featuring a fascinating interplay between *analysis*, *topology*, and *differential geometry*. The talk is intended to be without technicality, with essentially no proofs, and references will be given all along the talk, for anyone interested to further discover this topic.

Ladislav Drážný

Charles University

Reduction principle for potential convolution operators

Abstract: In this talk we will investigate potential convolution operators with nonnegative radially nonincreasing kernels. Let us consider a kernel K on $\mathbb{R}^n \setminus \{0\}$ and the respective potential convolution operator $Tf(x) = K * f(x) = \int_{\mathbb{R}^n} f(y)K(x-y)d\mu(y)$. Operators of this type include the classical Riesz potential. We will present here the reduction principle for these operators. In other words we will present the optimal conditions under which the convolution operator $T: X(\mathbb{R}^n, \mu) \rightarrow Y(\mathbb{R}^n, \nu)$ is bounded as the operator between two general rearrangement-invariant function spaces X, Y defined on measure spaces with different measures. We will also discuss compactness of these convolution operators.

Michał Dymek

Warsaw University of Technology

Continuous embeddings of variable exponent Besov and Triebel-Lizorkin spaces on metric measure spaces

Abstract: In this talk, we will discuss the characterization of continuous Sobolev embeddings of variable exponent Hajlasz-Besov and Hajlasz-Triebel-Lizorkin spaces in terms of the lower bound of the measure. Besov and Triebel-Lizorkin spaces provide natural scales of spaces that include a variety of function spaces used in analysis such as Lebesgue spaces, Sobolev spaces, Hardy spaces, Hölder spaces and BMO. They have retained their significance to this very day, playing an important role in both theoretical and applied branches of mathematics. We shall discuss the sufficient and necessary conditions for Sobolev-Poincaré inequalities on balls and for global Sobolev embeddings. Some results are new even in the constant exponent case. The talk is based on joint work with Ryan Alvarado, Przemysław Górka and Nijjwal Karak.

Nikita Evseev

Okinawa Institute of Science and Technology

Weak derivatives and metric differentiability almost everywhere

Abstract: It is known that a Lipschitz continuous map from the Euclidean domain to a metric space is metrically differentiable almost everywhere. When the metric space is a Banach space dual to separable, the metric differential has its linear counterpart – weak* differential. But for an arbitrary metric or Banach space, a Lipschitz map is not necessarily weak* differentiable. In this work, we suggest an approach based on weak weak* derivatives. In particular, it provides a linear representation, meaning that we can calculate the value of the metric differential as a norm of some linear operator.

Amiran Gogatishvili

Institute of Mathematics of the Czech Academy of Sciences

Reduction theorems for discrete Hardy operator on the cones of monotone sequences

Abstract: We present the discrete analogs of the reduction theorems of Hardy op-

erators restricted on the cone of non-negative, monotone sequences of the results for continuous case by Gogatishvili and Stepanov (see [1], [2]).

There is a difference between discrete and continuous cases. The proof in the discrete case does not work directly, as the power rule theorem, which is true in the continuous case, is not true in the discrete case. We also show that the precise formulation of the result from [1] is not true in discrete case.

References

- [1] A.Gogatishvili and V.D.Stepanov, *Reduction theorems for operators on the cones of monotone functions*. J. Math. Anal. Appl. 405 (2013), no. 1, 156-172.
- [2] A.Gogatishvili and V.D.Stepanov, *Reduction theorems for weighted integral inequalities on the cone of monotone functions*. Uspekhi Mat. Nauk 68 (2013), no. 4(412), 3-68. Russian Math. Surveys 68 (2013), no. 4, 597-664.

Adam Grzela

University of Warsaw

Minimizing fractional harmonic maps in homotopy classes from \mathbb{S}^3 to \mathbb{S}^2

Abstract: In 1998 T. Rivière proved that there exist infinitely many homotopy classes of $\pi_3(\mathbb{S}^2)$ having a minimizing 3-harmonic map. This result is especially surprising taking into account that in $\pi_3(\mathbb{S}^3)$ there are only three homotopy classes (corresponding to the degrees $\{-1, 0, 1\}$) in which a minimizer exists.

We extend this theorem in the framework of fractional harmonic maps and prove that for $s \in (0, 1)$ there exist infinitely many homotopy classes of $\pi_3(\mathbb{S}^2)$ in which there is a minimizing $W^{s, \frac{3}{s}}$ -harmonic map.

Gowri Sankara Raju Kosuru

Indian Institute of Technology Ropar

Trace principle for Riesz potentials on Herz spaces

Abstract: We establish trace inequalities for Riesz potentials on Herz-type spaces and discuss the optimality of conditions imposed on specific parameters. We also present some applications in the form of Sobolev-type inequalities, including the Gagliardo–Nirenberg–Sobolev inequality and the fractional integration theorem within the Herz space framework.

David Kubíček

Charles University

Interpolation between Lorentz spaces

Abstract: We briefly summarize known results concerning interpolation theory in the setting of Lorentz spaces and mention how classical operators behave in this setting. Finally we recall standard Calderón’s interpolation theorem about operators of joint weak type on rearrangement-invariant spaces and explore its generalization for the operators exhibiting boundedness on non-standard end-point Lorentz spaces.

Kaushik Mohanta

Charles University

Limits at infinity for functions in fractional Sobolev spaces

Abstract: We shall talk about the existence and non-existence of "limits at infinity" for functions in fractional Sobolev spaces. Depending on how we approach infinity, we shall give optimal ranges for parameters, for which limits do and do not exist.

The talk is based on a joint work with Angha Agarwal and Pekka Koskela.

Guillaume Neuttiens

Friedrich-Schiller Universität Jena

An introduction to the Newton Polygon method for time-periodic PDE's

Abstract: We present a straightforward method to study time-periodic solutions of some PDE's in an L^p setting. This method is twofold. On one hand, the use of time-averaging projections allows us to get rid of eventual singularities of the Fourier symbol at the origin (J. Sauer, M. Kyed). On the other hand, the theory of Newton polygons (L. Volevich, R. Denk) gives us some handful two-sided estimates to study the boundedness of the associated Fourier multipliers.

Dalimil Peša

Technische Universität Chemnitz and Charles University

Absolute continuity of the (quasi)norm in rearrangement-invariant spaces

Abstract: This talk explores the interactions of absolute continuity of the (quasi)norm with the concepts that are fundamental in the theory of rearrangement-invariant (quasi-)Banach function spaces, such as the Luxemburg representation or the Hardy–Littlewood–Pólya relation. Specifically, we show that the absolute continuity of the (quasi)norm in a given rearrangement-invariant (quasi-)Banach function space corresponds naturally to the same concept in its representation space, provided that this representation space is constructed in a suitable way (an example of which we present). Further, we show that when the space satisfies the Hardy–Littlewood–Pólya principle, then the subspace of functions having absolutely continuous (quasi)norm is an order ideal with respect to the Hardy–Littlewood–Pólya relation.

Paul Stephan

University of Konstanz

Korn-Maxwell-Sobolev inequalities for constant-rank operators

Abstract: We present Korn-Maxwell-Sobolev (KMS) inequalities for operators of constant rank. These generalize Korn inequalities for incompatible matrix fields and extend prior work (by Gmeineder, Lewintan and Neff) that focused on elliptic operators. The key to this extension to the constant-rank setting is the introduction of a specific correction term. Joint work with Peter Lewintan.

Durvudkhan Suragan

Nazarbayev University

Hardy-Leray type inequalities in variable Lebesgue spaces

Abstract: In this talk, we discuss the problem of extending the classical Hardy-Leray inequality and related inequalities to the scale of variable Lebesgue spaces. This is joint work with David Cruz-Urbe (University of Alabama).

Leon Winter

UCLouvain

Morrey–Sobolev-type inequalities in weighted Sobolev spaces

Abstract: The well-known Morrey–Sobolev inequality, due to C.B. Morrey (1938), ensures that, if $n = \dim \Omega < p$, functions in the homogeneous Sobolev space $\dot{W}^{1,p}(\Omega)$ are actually Hölder-continuous *up to the boundary* of Ω with exponent $\alpha = 1 - \frac{n}{p}$. However, when considering (homogeneous) *weighted Sobolev spaces* $\dot{W}_\gamma^{1,p}(\Omega)$, which consist of weakly differentiable functions such that

$$\int_{\Omega} |Du(x)|^p \operatorname{dist}(x, \partial\Omega) \, dx < +\infty,$$

the regularity *up to the boundary* depends on the parameter $\gamma \in \mathbb{R}$.

In this talk, I will present some new Morrey–Sobolev-type inequalities for these spaces, which give a full overview of the interplay between p , $\dim \Omega$, and γ , as well as the *best possible* Hölder-type continuity for $\dot{W}_\gamma^{1,p}(\Omega)$ functions.

Posters / Offprints

Ondřej Bouchala

Czech Technical University in Prague

Weak limit of $W^{1,2}$ homeomorphisms in \mathbb{R}^3 can have any degree

Abstract: In this paper, for every $k \in \mathbb{Z}$, we construct a sequence of homeomorphisms $h_m: B(0, 10) \rightarrow \mathbb{R}^3$ that converge weakly to a map h in $W^{1,2}(B(0, 10))$, such that $h_m(x) = x$ on $\partial B(0, 10)$, and for every $r \in (\frac{5}{16}, \frac{7}{16})$, the degree of h with respect to the ball $B(0, r)$ is equal to k on a set of positive measure.

Authors: Ondřej Bouchala, Stanislav Hencl and Zheng Zhu.

Daniel Campbell

Charles University

Diffeomorphic approximation of piecewise affine homeomorphisms

Abstract: Given any f a locally finitely piecewise affine homeomorphism of $\Omega \subset \mathbb{R}^d$ onto $\Delta \subset \mathbb{R}^d$ (for $d = 3, 4$) such that $f \in W^{1,p}(\Omega, \mathbb{R}^d)$ and $f^{-1} \in W^{1,q}(\Delta, \mathbb{R}^d)$, $1 \leq p, q < \infty$ and any $\epsilon > 0$ we construct a diffeomorphism \tilde{f} such that

$$\|f - \tilde{f}\|_{W^{1,p}(\Omega, \mathbb{R}^d)} + \|f^{-1} - \tilde{f}^{-1}\|_{W^{1,p}(\Delta, \mathbb{R}^d)} < \epsilon.$$

Lukas Fußangel

University of Konstanz

On the singular set of BV-minimizers for non-autonomous functionals

Abstract: In the article exhibited, we investigate regularity properties of minimizers for non-autonomous convex variational integrands $F(x, Du)$ with linear growth, defined on bounded Lipschitz domains $\Omega \subset \mathbb{R}^n$. Linear growth functionals arise e.g. in the study of minimal surfaces or plasticity and are therefore an important class of variational problems.

Assuming appropriate degenerate ellipticity and Hölder continuity with respect to the first variable, we show that gradients of minimizers have higher integrability. We also provide bounds on the Hausdorff dimension of the singular set of minimizers.

Authors: Lukas Fußangel, Buddhika Priyasad and Paul Stephan.

Christopher Körber

Charles University

A logarithmically bounded number of small rigid bodies in a viscous incompressible inhomogeneous fluid is negligible

Abstract: We consider a large number of inhomogeneous rigid bodies immersed in an inhomogeneous incompressible fluid contained in a bounded domain of dimension bigger or equal to two. We address the question about the asymptotic behaviour of the corresponding system of partial differential equations as the number of rigid bodies tends to infinity with a logarithmic bound and the diameter of the bodies tends to zero. We show that the rigid bodies are neglected in the limit in the sense that the limit system is given only by the inhomogeneous

incompressible Navier-Stokes System. We do not require any regularity for the boundaries of the rigid bodies and the domain.

In order to pass to the limit in the nonlinear term of the Navier-Stokes Equations, we prove a compactness result in the Bochner Space $L^p(S, X)$, which may be of independent interest.

Our result extends earlier work by Feireisl, Roy and Zarnescu (2023) to more general assumptions which are of physical relevance. This includes assumptions on the mass densities of the fluid and the rigid bodies, which are allowed to be inhomogeneous, can attain the value zero, and only need to be bounded in some suitable L^p -space instead of being uniformly bounded.

Martin Křepela

Czech Technical University in Prague

Homogeneity of rearrangement-invariant norms

Abstract: We study rearrangement-invariant spaces X over $[0, \infty)$ for which there exists a function $h : (0, \infty) \rightarrow (0, \infty)$ such that

$$\|D_r f\|_X = h(r)\|f\|_X$$

for all $f \in X$ and all $r > 0$, where D_r is the dilation operator. It is shown that this may hold only if $h(r) = r^{-\frac{1}{p}}$ for all $r > 0$, in which case the norm $\|\cdot\|_X$ is called p -homogeneous. We investigate which types of r.i. spaces satisfy this condition and show some important embedding properties.

Authors: Santiago Boza, Martin Křepela and Javier Soria.

Zdeněk Mihula

Czech Technical University in Prague

Compact Sobolev embeddings of radially symmetric functions

Abstract: We provide a complete characterization of compactness of Sobolev embeddings of radially symmetric functions on the entire space \mathbb{R}^n in the general framework of rearrangement-invariant function spaces. We avoid any unnecessary restrictions and cover also embeddings of higher order, providing a complete picture within this framework. To achieve this, we need to develop new techniques because the usual techniques used in the study of compactness of Sobolev embeddings in the general framework of rearrangement-invariant function spaces are limited to domains of finite measure, which is essential for them to work. Furthermore, we also study certain weighted Sobolev embeddings of radially symmetric functions on balls. We completely characterize their compactness and also describe optimal target rearrangement-invariant function spaces in these weighted Sobolev embeddings.

Farah Alissa Binti Mislal

University of Salerno

Anisotropic operators in generalized Morrey spaces

Abstract: In this work, we investigate the boundedness and continuity properties of anisotropic sublinear operators of Calderón-Zygmund type, anisotropic Riesz potentials, and fractional maximal operators within the framework of vanishing generalized Morrey spaces [1]. These spaces provide a flexible setting to cap-

ture fine local behavior of functions in non-homogeneous and directionally scaled contexts, relevant to anisotropic harmonic analysis [2].

We introduce a broad class of growth functions that satisfy appropriate structural and integrability conditions. Under these assumptions, we establish that the considered operators are bounded from the vanishing generalized Morrey space $V\mathcal{M}^{p,\varphi(\cdot)}(\mathbb{R}^n)$ into itself or into related function spaces [1], [3]. Furthermore, we prove that the space $V\mathcal{M}^{p,\varphi(\cdot)}(\mathbb{R}^n)$ is complete and that smooth compactly supported functions form a dense subset [4], [5], ensuring the robustness of the functional analytic framework.

Our approach is based on deriving precise pointwise modular estimates that control the operator behavior at small scales. These estimates allow for refined control over the action of singular and potential-type operators in anisotropic settings [6], [7]. The proofs rely on harmonic analysis techniques adapted to the vanishing Morrey space structure and anisotropic geometry [1], [8].

Authors: Farah Alissa Binti Mislal, Lyoubomira Softova and Paola Cavaliere.

References

- [1] N. Samko, *Potential and singular operators in vanishing generalized morrey spaces*. J. Global Optim, 2013, vol. 57, p. 1385–1399.
- [2] M. Bramanti and M.C. Cerutti, *Commutators of singular operators on homogeneous spaces*. Boll. Un. Mat. Ital. B, 2003, vol. II, pp. 843–883.
- [3] L. Softova, *Singular integrals and commutators in generalized morrey spaces*. Acta Math. Sin. (Engl. Ser.), 2006, vol. 22-3, p. 757–766.
- [4] C.T. Zorko, *Morrey space*. Proc. Of The Amer. Math. Soci., 1986, vol. 98, p. 586–592.
- [5] L. Softova L. Caso, R. D’Ambrosio, *Generalized Morrey spaces over unbounded domains*. Azerbaijan Journal of Mathematics, 2020, vol. 10.
- [6] E.B. Fabes and N.M. Rivière, *Commutators of singular integrals*. Studia Math, 1996, vol. 10-7, pp. 9–17.
- [7] D. Gilbarg and N.S. Trudinger, *Elliptic partial differential equations of second order*. Springer-Verlag, 1983.
- [8] E. Nakai, *Hardy-Littlewood maximal operator, singular integral operators and the riesz potentials on generalized Morrey spaces*. Math. Nachr, 1994, vol. 166, p. 95–103.

Luboř Pick

Charles University

On the modulus of continuity of fractional Orlicz-Sobolev functions

Abstract: Necessary and sufficient conditions are presented for a fractional Orlicz-Sobolev space on the Euclidean space \mathbb{R}^n to be continuously embedded into a space of uniformly continuous functions. The optimal modulus of continuity is exhibited whenever these conditions are fulfilled. These results pertain to the supercritical Sobolev regime and complement earlier sharp embeddings into rearrangement-invariant spaces concerning the subcritical setting. Classical embeddings for fractional Sobolev spaces into Hölder spaces are recovered as special instances. Proofs require novel strategies, since customary methods fail to produce optimal conclusions.

Authors: Angela Alberico, Andrea Cianchi, Luboř Pick and Lenka Slavíková.

Lenka Slavíková

Charles University

Strongly nonlinear Robin problems for harmonic and polyharmonic functions in the half-space

Abstract: Existence and global regularity results for boundary-value problems of Robin type for harmonic and polyharmonic functions in n -dimensional half-spaces are offered. The Robin condition on the normal derivative on the boundary of the half-space is prescribed by a nonlinear function \mathcal{N} of the relevant harmonic or polyharmonic functions. General Orlicz type growths for the function \mathcal{N} are considered. For instance, functions \mathcal{N} of classical power type, their perturbations by logarithmic factors, and exponential functions are allowed. New sharp boundedness properties in Orlicz spaces of some classical operators from harmonic analysis, of independent interest, are critical for our approach. This is a joint work with Andrea Cianchi and Gael Y. Diebou.

Hana Turčinová

Czech Technical University in Prague

On the properties of rearrangement-invariant quasi-Banach function spaces

Abstract: This paper explores some important aspects of the theory of rearrangement-invariant quasi-Banach function spaces. We focus on two main topics. Firstly, we prove an analogue of the Luxemburg representation theorem for rearrangement-invariant quasi-Banach function spaces over resonant measure spaces. Secondly, we develop the theory of fundamental functions and endpoint spaces.

Authors: Anna Musilová, Aleš Nekvinda, Dalimil Peša and Hana Turčinová.