# Spring School on Analysis 2025: Function Spaces and Applications XIII

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Ondřej Bouchala

*Czech Technical University in Prague*

### Weak limit of $W^{1,2}$ homeomorphisms in $\mathbb{R}^3$ can have any degree

Abstract: In this paper, for every $k \in \mathbb{Z}$, we construct a sequence of homeomorphisms $h_m \colon B(0,10) \to \mathbb{R}^3$ that converge weakly to a map $h$ in $W^{1,2}(B(0,10))$, such that $h_m(x) = x$ on $\partial B(0,10)$, and for every $r \in \left(\frac{5}{16}, \frac{7}{16}\right)$, the degree of $h$ with respect to the ball $B(0,r)$ is equal to $k$ on a set of positive measure.
Authors: Ondřej Bouchala, Stanislav Hencl and Zheng Zhu.

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Daniel Campbell

*Charles University*

### Diffeomorphic approximation of piecewise affine homeomorphisms

Abstract: Given any $f$ a locally finitely piecewise affine homeomorphism of $\Omega \subset \mathbb{R}^d$ onto $\Delta \subset \mathbb{R}^d$ (for $d = 3, 4$) such that $f \in W^{1,p}(\Omega, \mathbb{R}^d)$ and $f^{-1} \in W^{1,q}(\Delta, \mathbb{R}^d)$, $1 \leq p, q < \infty$ and any $\epsilon > 0$ we construct a diffeomorphism $\widetilde{f}$ such that

$$\|f - \widetilde{f}\|_{W^{1,p}(\Omega,\mathbb{R}^d)} + \|f^{-1} - \widetilde{f}^{-1}\|_{W^{1,p}(\Delta,\mathbb{R}^d)} < \epsilon.$$

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Lukas Fußangel

*University of Konstanz*

### On the singular set of $\mathrm{BV}$-minimizers for non-autonomous functionals

Abstract: In the article exhibited, we investigate regularity properties of minimizers for non-autonomous convex variational integrands $F(x, Du)$ with linear growth, defined on bounded Lipschitz domains $\Omega \subset \mathbb{R}^n$. Linear growth functionals arise e.g. in the study of minimal surfaces or plasticity and are therefore an important class of variational problems.
Assuming appropriate degenerate ellipticity and Hölder continuity with respect to the first variable, we show that gradients of minimizers have higher integrability. We also provide bounds on the Hausdorff dimension of the singular set of minimizers.
Authors: Lukas Fußangel, Buddhika Priyasad and Paul Stephan.

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Christopher Körber

*Charles University*

### A logarithmically bounded number of small rigid bodies in a viscous incompressible inhomogeneous fluid is negligible

Abstract: We consider a large number of inhomogeneous rigid bodies immersed in an inhomogeneous incompressible fluid contained in a bounded domain of dimension bigger or equal to two. We address the question about the asymptotic behaviour of the corresponding system of partial differential equations as the number of rigid bodies tends to infinity with a logarithmic bound and the diameter of the bodies tends to zero. We show that the rigid bodies are neglected in the limit in the sense that the limit system is given only by the inhomogeneous incompressible Navier-Stokes System. We do not require any regularity for the boundaries of the rigid bodies and the domain.
In order to pass to the limit in the nonlinear term of the Navier-Stokes Equations, we prove a compactness result in the Bochner Space $L^p(S, X)$, which may be of independent interest.
Our result extends earlier work by Feireisl, Roy and Zarnescu (2023) to more general assumptions which are of physical relevance. This includes assumptions on the mass densities of the fluid and the rigid bodies, which are allowed to be inhomogeneous, can attain the value zero, and only need to be bounded in some suitable $L^p$-space instead of being uniformly bounded.

Martin Křepela

*Czech Technical University in Prague*

### Homogeneity of rearrangement-invariant norms

Abstract: We study rearrangement-invariant spaces $X$ over $[0, \infty)$ for which there exists a function $h : (0, \infty) \to (0, \infty)$ such that

$$\|D_r f\|_X = h(r)\|f\|_X$$

for all $f \in X$ and all $r > 0$, where $D_r$ is the dilation operator. It is shown that this may hold only if $h(r) = r^{-\frac{1}{p}}$ for all $r > 0$, in which case the norm $\|\cdot\|_X$ is called $p$-homogeneous. We investigate which types of r.i. spaces satisfy this condition and show some important embedding properties.
Authors: Santiago Boza, Martin Křepela and Javier Soria.

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Zdeněk Mihula

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### Compact Sobolev embeddings of radially symmetric functions

Abstract: We provide a complete characterization of compactness of Sobolev embeddings of radially symmetric functions on the entire space $\mathbb{R}^n$ in the general framework of rearrangement-invariant function spaces. We avoid any unnecessary restrictions and cover also embeddings of higher order, providing a complete picture within this framework. To achieve this, we need to develop new techniques because the usual techniques used in the study of compactness of Sobolev embeddings in the general framework of rearrangement-invariant function spaces are limited to domains of finite measure, which is essential for them to work. Furthermore, we also study certain weighted Sobolev embeddings of radially symmetric functions on balls. We completely characterize their compactness and also describe optimal target rearrangement-invariant function spaces in these weighted Sobolev embeddings.

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Luboš Pick

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### On the modulus of continuity of fractional Orlicz-Sobolev functions

Abstract: Necessary and sufficient conditions are presented for a fractional Orlicz-Sobolev space on the Euclidean space $\mathbb{R}^n$ to be continuously embedded into a space of uniformly continuous functions. The optimal modulus of continuity is exhibited whenever these conditions are fulfilled. These results pertain to the supercritical Sobolev regime and complement earlier sharp embeddings into rearrangement-invariant spaces concerning the subcritical setting. Classical embeddings for fractional Sobolev spaces into Hö lder spaces are recovered as special instances. Proofs require novel strategies, since customary methods fail to produce optimal conclusions.
Authors: Angela Alberico, Andrea Cianchi, Luboš Pick and Lenka Slavíková.

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Lenka Slavíková

*Charles University*

### Strongly nonlinear Robin problems for harmonic and polyharmonic functions in the half-space

Abstract: Existence and global regularity results for boundary-value problems of Robin type for harmonic and polyharmonic functions in $n$-dimensional half-spaces are offered. The Robin condition on the normal derivative on the boundary of the half-space is prescribed by a nonlinear function $\mathcal{N}$ of the relevant harmonic or polyharmonic functions. General Orlicz type growths for the function $\mathcal{N}$ are considered. For instance, functions $\mathcal{N}$ of classical power type, their perturbations by logarithmic factors, and exponential functions are allowed. New sharp boundedness properties in Orlicz spaces of some classical operators from harmonic analysis, of independent interest, are critical for our approach. This is a joint work with Andrea Cianchi and Gael Y. Diebou.

Hana Turčinová

*Czech Technical University in Prague*

### On the properties of rearrangement-invariant quasi-Banach function spaces

Abstract: This paper explores some important aspects of the theory of rearrangement-invariant quasi-Banach function spaces. We focus on two main topics. Firstly, we prove an analogue of the Luxemburg representation theorem for rearrangement-invariant quasi-Banach function spaces over resonant measure spaces. Secondly, we develop the theory of fundamental functions and endpoint spaces.
Authors: Anna Musilová, Aleš Nekvinda, Dalimil Peša and Hana Turčinová.

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Farah Alissa Binti Mislar

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### Anisotropic operators in generalized Morrey spaces

Abstract: In this work, we investigates the boundedness and continuity properties of anisotropic sublinear operators of Calderón-Zygmund type, anisotropic Riesz potentials, and fractional maximal operators within the framework of vanishing generalized Morrey spaces [1]. These spaces provide a flexible setting to capture fine local behavior of functions in non-homogeneous and directionally scaled contexts, relevant to anisotropic harmonic analysis [2].

We introduce a broad class of growth functions that satisfy appropriate structural and integrability conditions. Under these assumptions, we establish that the considered operators are bounded from the vanishing generalized Morrey space $V\mathcal{M}^{p,\varphi(\cdot)}(\mathbb{R}^n)$ into itself or into related function spaces [1], [3]. Furthermore, we prove that the space $V\mathcal{M}^{p,\varphi(\cdot)}(\mathbb{R}^n)$ is complete and that smooth compactly supported functions form a dense subset [4], [5], ensuring the robustness of the functional analytic framework.

Our approach is based on deriving precise pointwise modular estimates that control the operator behavior at small scales. These estimates allow for refined control over the action of singular and potential-type operators in anisotropic settings [6], [7]. The proofs rely on harmonic analysis techniques adapted to the vanishing Morrey space structure and anisotropic geometry [1], [8].
Authors: Farah Alissa Binti Mislar, Lyoubomira Softova and Paola Cavaliere.

### References

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